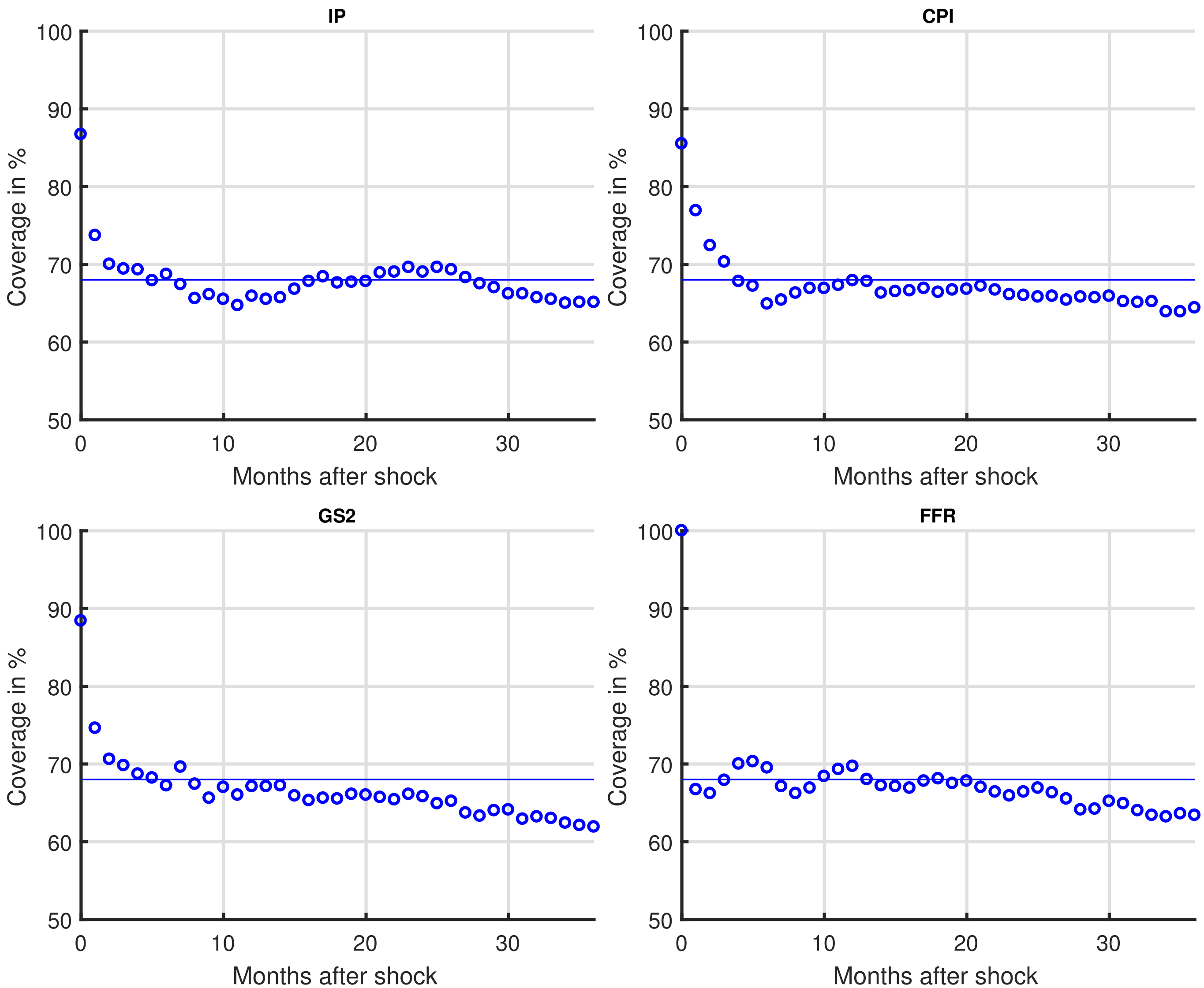
APPENDIX B: ADDITIONAL TABLES AND FIGURES

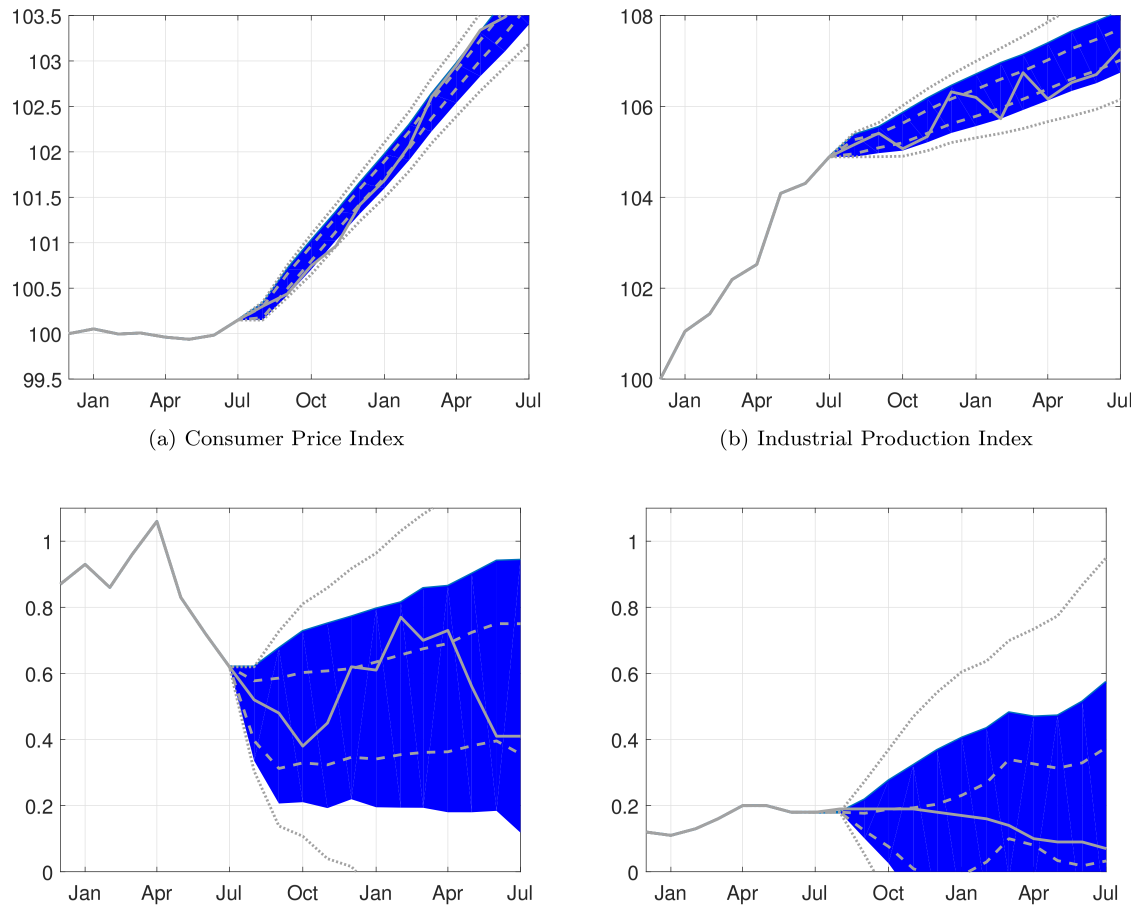
B.1. *Additional Tables and Figures*

Figure 5: Robust Bayesian credibility of the delta-method interval based on the posterior distribution $\mu^* \sim \mathcal{N}(\widehat{\mu}_T, \widehat{\Omega}_T/T)$, $T = 342$.



(Circles) Monte-Carlo estimate of the probability $P^*_\mu\Big([\underline{v}_{k,i,j}(\mu^*), \overline{v}_{k,i,j}(\mu^*)] \subset [\underline{v}_{k,i,j}(\widehat{\mu}_T) - .9945\widehat{\sigma}_{(k,i,j),T}/\sqrt{T}, \overline{v}_{k,i,j}(\widehat{\mu}_T) + .9945\widehat{\sigma}_{(k,i,j),T}/\sqrt{T} | Y_1, \ldots, Y_T]\Big)$ for the posterior distribution $\mu^* \sim \mathcal{N}(\widehat{\mu}_T, \widehat{\Omega}_T)$, with $T = 342$. The values $\widehat{\mu}_T$ and $\widehat{\Omega}_T$ correspond, respectively, to the estimators of the reduced-form parameter and its asymptotic covariance matrix in the UMP application. (Solid Line) Nominal credibility for the delta-method confidence interval (68%).

Figure 6: Projection Confidence Interval for CPI, IP, 2yTB, FF after the August 2010 announcement



(a) Consumer Price Index

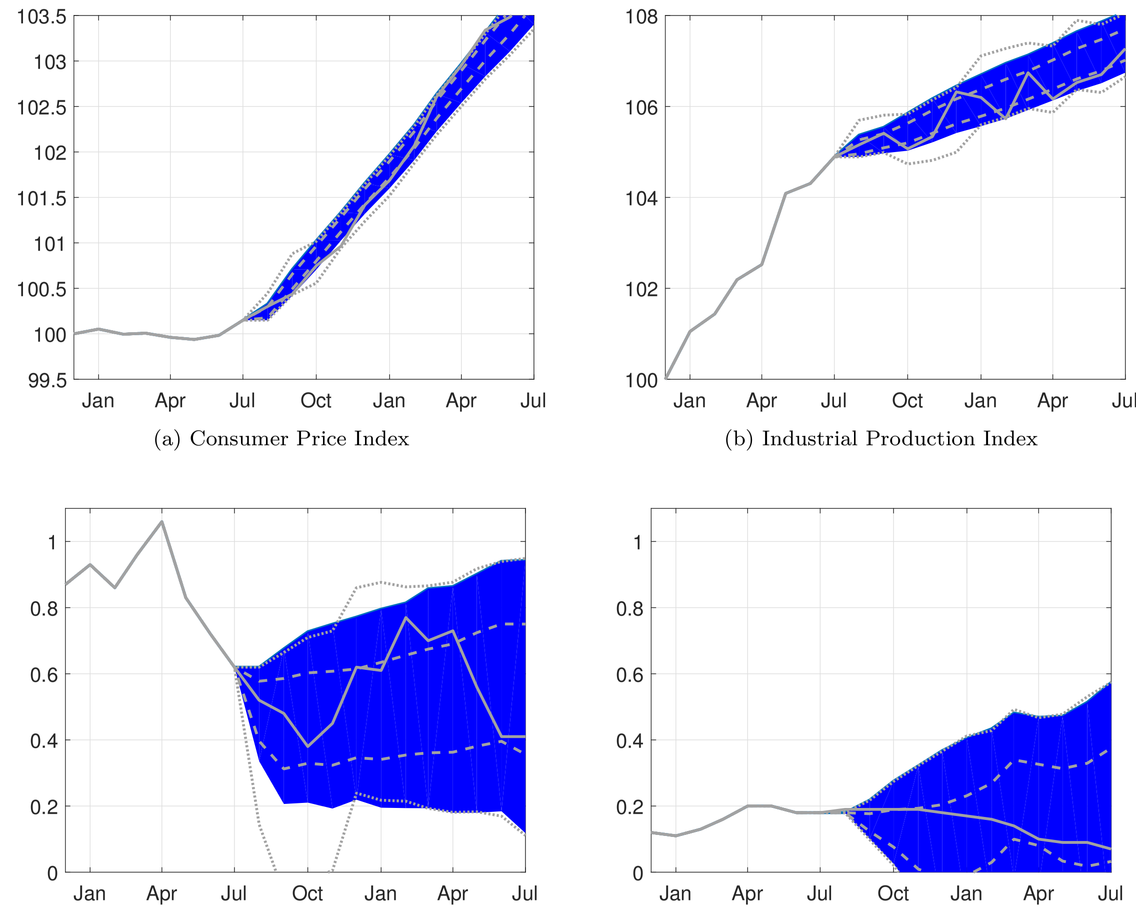
(b) Industrial Production Index

(c) 2-year Treasury Bond Rate

(d) Federal Funds Rate

(Shaded Area) Evolution of the Levels CPI, IP, 2yTB, and FF based on our 68% delta method confidence bands for the coefficients of Cumulative Impulse-Response Functions. (Solid Line) Observed Levels of CPI, IP, 2yTB, and FF from December 2009 to July 2011. Both the CPI index and the IP index were normalized to have a starting value of 100. (Dashed Line) 68% credible set constructed using the priors in Uhlig (2005). (Gray, Dotted Line) Gafarov et al. (2016)'s 68% confidence interval based on the projection approach.

Figure 7: Robust Credible Set for CPI, IP, 2yTB, FF after the August 2010 announcement



(a) Consumer Price Index

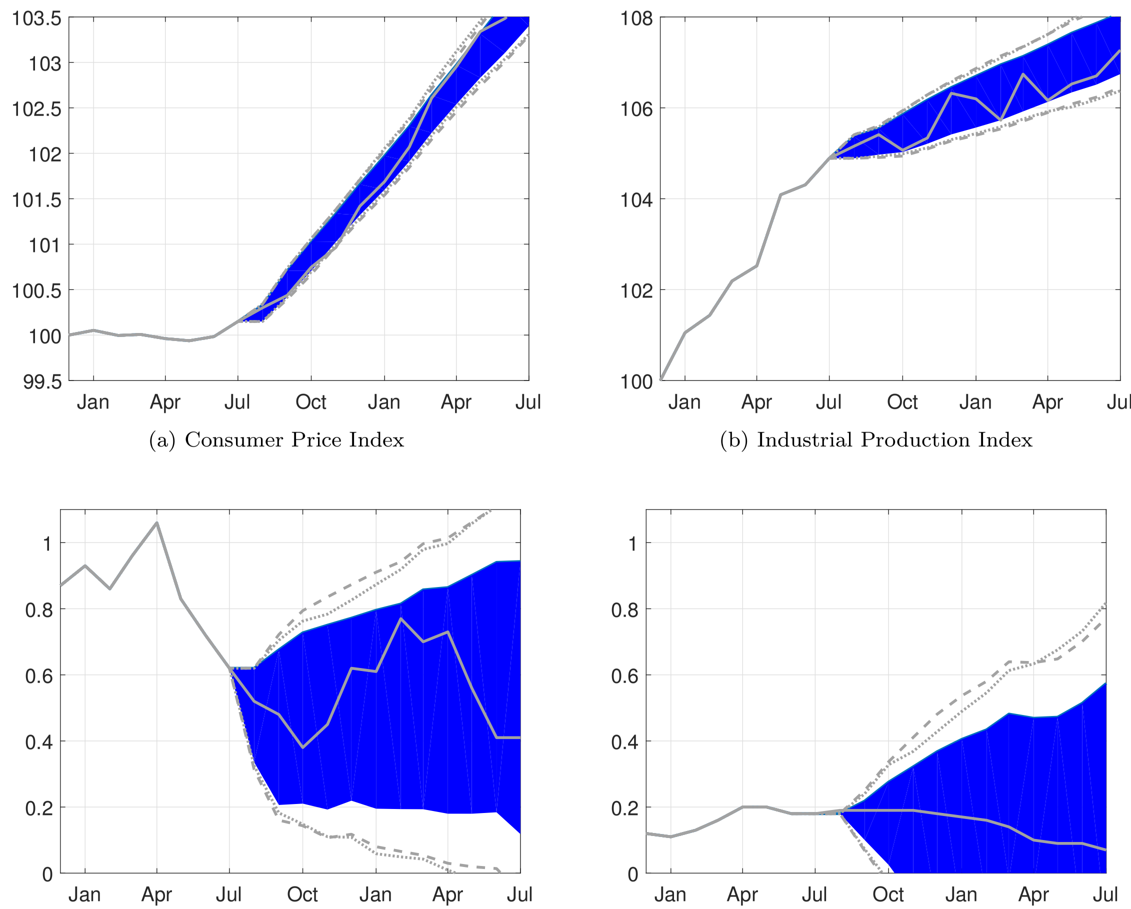
(b) Industrial Production Index

(c) 2-year Treasury Bond Rate

(d) Federal Funds Rate

(Shaded Area) Evolution of the Levels CPI, IP, 2yTB, and FF based on our 68% delta method confidence bands for the coefficients of Cumulative Impulse-Response Functions. (Solid Line) Observed Levels of CPI, IP, 2yTB, and FF from December 2009 to July 2011. Both the CPI index and the IP index were normalized to have a starting value of 100. (Dashed Line) 68% credible set constructed using the priors in Uhlig (2005). (Dotted Line) Giacomini and Kitagawa (2015)'s 68% robust-Bayes credible set constructed using the priors for the reduced-form parameters in Uhlig (2005)

Figure 8: Joint Credible Set (corresponding to all 4 variables and 36 horizons) for impulse response functions of CPI, IP, 2yTB, FF after the August 2010 announcement



(a) Consumer Price Index

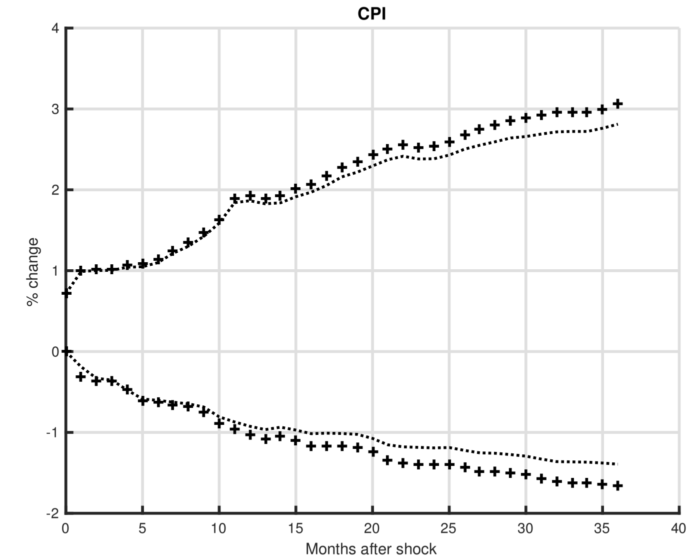
(b) Industrial Production Index

(c) 2-year Treasury Bond Rate

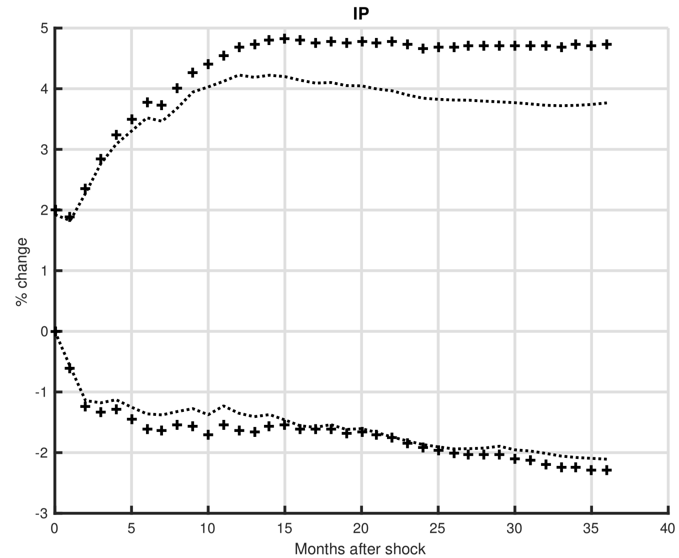
(d) Federal Funds Rate

(Shaded Area) Evolution of the Levels CPI, IP, 2yTB, and FF based on our 68% delta method confidence bands for the coefficients of Cumulative Impulse-Response Functions. (Solid Line) Observed Levels of CPI, IP, 2yTB, and FF from December 2009 to July 2011. Both the CPI index and the IP index were normalized to have a starting value of 100. (Dashed Line) Bonferroni-corrected joint 68% delta method confidence bands. (Dotted Line) Inoue and Kilian (2013)'s joint 68% Bayes credible set for impulse response functions using the priors for the reduced-form parameters in Uhlig (2005).

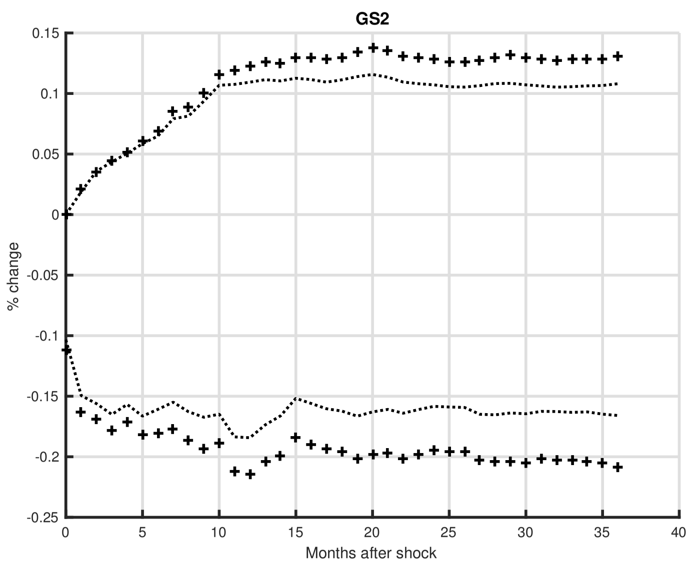
Figure 9: Confidence set for impulse response functions of CPI, IP, 2yTB, FF to a UMP shock under alternative identification scheme



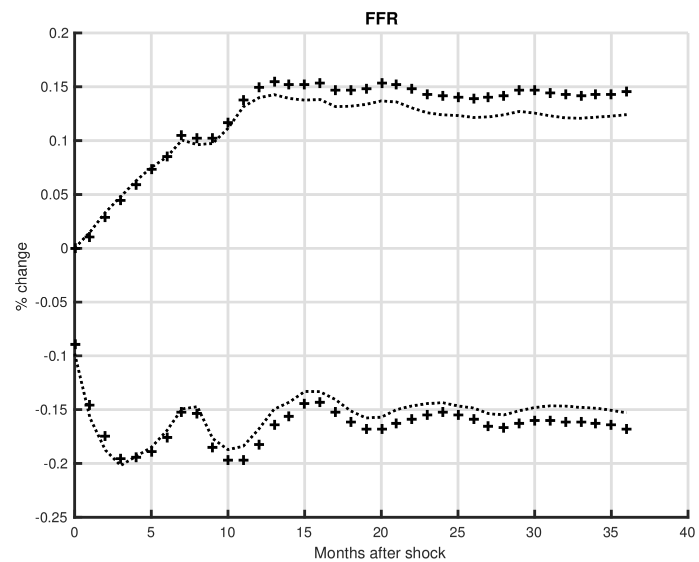
(a) Consumer Price Index



(b) Industrial Production Index



(c) 2-year Treasury Bond Rate



(d) Federal Funds Rate

(Pluses) 68% Bonferroni-type confidence bounds of Granziera et al. (2017). See details on implementation in A.7.1 . (Dots) 68% delta method confidence bands

Note: the identification scheme used to produce these plots differs from the one used for Figure 1. The zero restriction on the response of the FFR is replaced with a negative sign restriction to improve the acceptance rate of the algorithm for Bonferroni-type CS.

Figure 10: The mathematical program defining $\overline{v}_{k,i,j}(\mu)$ $(n = 3)$ with one zero restriction.

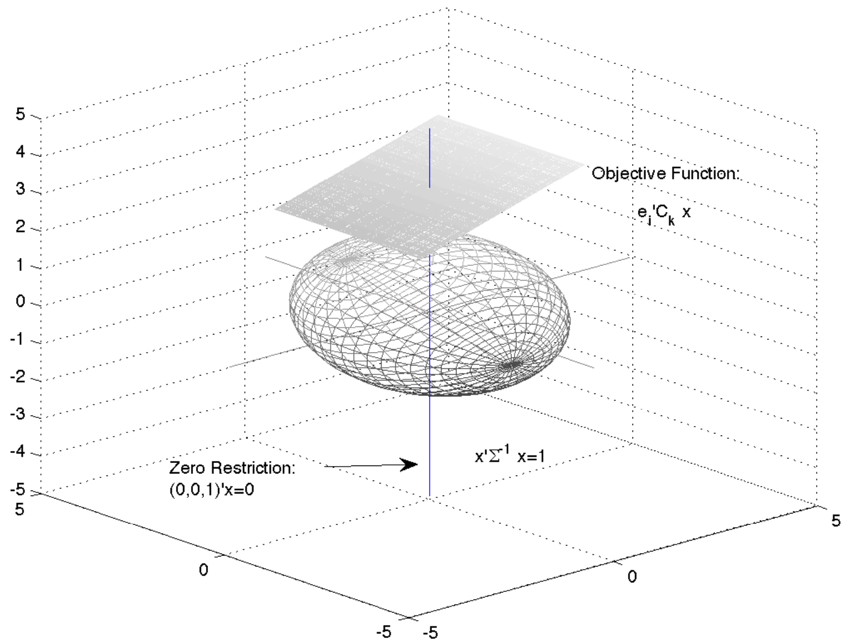


Figure 10 provides a graphical representation of the mathematical program (2.5), where $BB' = \Sigma$ has been replaced by the 'ellipsoid' constraint $x'\Sigma^{-1}x = 1$, $x \equiv B_j \in \mathbb{R}^3$. The objective function corresponds to the hyperplane with the normal vector $C_k(A)'e_i \in \mathbb{R}^3$. In this example, there is only one equality restriction with the normal vector given by the solid line. This restriction requires the contemporaneous impact of the j-th shock on the third variable to be zero. Note that without the equality restriction the maximizer and minimizer will be given by the point at which the hyperplane is tangent to the ellipsoid.

Figure 11: Solving for $\overline{v}_{k,i,j}(\mu)$ ($n = 3$, $\Sigma = \mathbb{I}_3$) with one equality restriction.

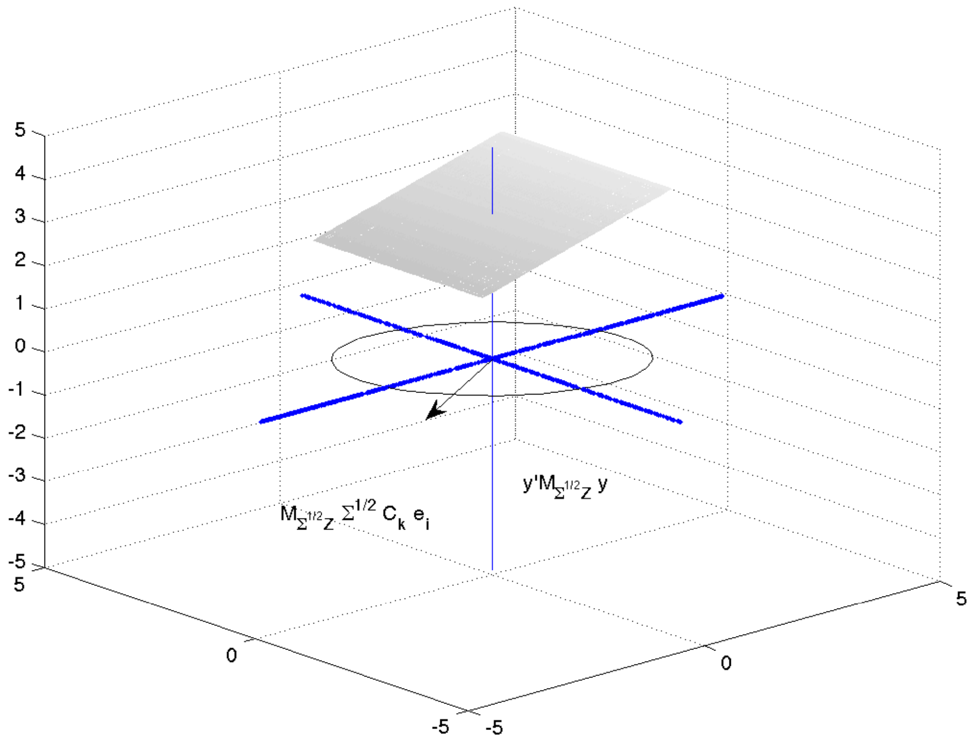


Figure 11 provides a graphical representation of the solution to the mathematical program (2.5) when $\Sigma = \mathbb{I}_3$ and there is only one zero restriction. The solution to the program must lie in the orthogonal complement of Z (the thin solid line). In this picture, the orthogonal complement corresponds to the space spanned by the thick solid lines. This implies that the rotated solution, denoted $\widetilde{x} \equiv \Sigma^{-1/2}x$, must be of the form $M_{\Sigma^{1/2}Z}y$ for some $y \in \mathbb{R}^3$. Hence, the only relevant part of $x'\Sigma^{-1}x = 1$ becomes the projected version of it: $y'M_{\Sigma^{1/2}}y = 1$, represented by the ellipsoid. One can find the value of this problem by projecting the gradient of the objective function on the orthogonal complement of $\Sigma^{1/2}z$ (the arrow) and selecting a direction in the ellipsoid collinear to it. The value function $\overline{v}_{k,ij}(\mu)$ will be given by the norm of the arrow.

Suppose there are only equality constraints. Note that $Z'B_j = \mathbf{0}_{m\times 1}$ implies that the re-parameterized choice variable $\widetilde{x} \equiv \Sigma^{-1/2}B_j$ must lie on the orthogonal space of $\Sigma^{1/2}Z$. That is, the selected value of $\widetilde{x}$ should be of the form

$$\widetilde{x} = M_{\Sigma^{1/2}Z}y, \quad M_{\Sigma^{1/2}Z} \equiv \left(\mathbb{I}_n - \Sigma^{1/2}Z(Z'\Sigma Z)^{-1}Z'\Sigma^{1/2}\right), \ y \in \mathbb{R}^n.$$

The quadratic equality constraint also restricts the choice variable $\widetilde{x}$ to satisfy $\widetilde{x}'\widetilde{x} = 1$. Consequently, the problem can be re-written as

$$\max_{y\in\mathbb{R}^n} e_i'C_k\Sigma^{1/2}M_{\Sigma^{1/2}Z}y \quad \text{s.t.} \quad y'M_{\Sigma^{1/2}Z}y = 1.$$

An application of the Cauchy-Schwartz inequality shows that the positive value in $(A.1)$ gives the maximum response in (2.5).